PART 2

Number properties and divisibility

3 sets · 30 questions · 6 more in Under the clock

Set 4 Primes and factors, LCM/GCF and factor counts

10 questions · target time 27:15

1 HARD

How many positive factors of 10,800 are multiples of 12 but are not multiples of 9?

A 3 **B** 6 **C** 9 **D** 18 **E** 27

2 HARD

What is the greatest integer k such that 2^k is a factor of $5^{24} - 1$?

A 2 **B** 3 **C** 4 **D** 5 **E** 7

3 HARD

For how many positive integers n less than or equal to 200 is the greatest common factor of n and 72 equal to 8?

A 9 **B** 13 **C** 16 **D** 17 **E** 25

4 HARD

If a and b are positive integers and the greatest common factor of a and b is 12, which of the following could be the greatest common factor of a^2 and $3b$?

I. 72

II. 108

III. 144

A I only **B** II only **C** III only **D** I and II only **E** I and III only

5 HARD

If n is a positive integer and n^3 has exactly 28 positive factors, which of the following could be the number of positive factors of n^2?

I. 15

II. 18

III. 19

A I only **B** II only **C** III only **D** I and III only **E** I, II and III

6 HARD

The positive integer n has exactly 4 positive factors, and the sum of those 4 factors is 48. What is the sum of all possible values of n?

A 33 **B** 35 **C** 68 **D** 91 **E** 98

7 EXTREME

The positive integers a and b, with $a < b$, satisfy $a + b = 100$, and the least common multiple of a and b equals $a + b + g$, where g is the greatest common factor of a and b. What is the value of a?

A 20 **B** 25 **C** 40 **D** 50 **E** 60

8 EXTREME

The positive integers a, b and c are such that the greatest common factor of a and b is 6 and the greatest common factor of b and c is 10. Which of the following could be the greatest common factor of a and c?

I. 2

II. 6

III. 30

A I only **B** II only **C** III only **D** I and II only **E** I, II and III

9 EXTREME

For each positive integer n, let $L(n)$ be the least common multiple of the integers from 1 to n, inclusive. For how many integers n from 1 to 30, inclusive, is $L(n+1) = L(n)$?

A 11 **B** 13 **C** 15 **D** 17 **E** 19

10 EXTREME

For how many pairs of positive integers a and b with $a < b \leq 60$ is the least common multiple of a and b equal to 12 times their greatest common factor?

A 15 **B** 20 **C** 30 **D** 40 **E** 60

Set 5 Divisibility and remainders

10 questions · target time 28:50

1 HARD

When the positive integer n is divided by the positive integer d, the quotient is 7 and the remainder is 11. When $2n$ is divided by d, the remainder is 3. What is the value of n?

A 95 **B** 136 **C** 144 **D** 165 **E** 186

2 HARD

For how many integers n from 1 to 60, inclusive, is $n^2 + n + 1$ divisible by 3 or by 7?

A 18 **B** 20 **C** 26 **D** 32 **E** 38

3 HARD

When each of 58, 97 and 139 is divided by the two-digit positive integer d, the three remainders add up to exactly d. What is the value of d?

A 14 **B** 21 **C** 42 **D** 49 **E** 98

4 HARD

When the positive integer a is divided by 9, the remainder is 5, and when the positive integer b is divided by 6, the remainder is 4. Which of the following could be the remainder when ab is divided by 18?

I. 2

II. 8

III. 11

A I only
B I and II only
C I and III only
D II and III only
E I, II and III

5 HARD

What is the remainder when $2^{100} + 2^{70} + 2^{40}$ is divided by 1,025?

A 1 **B** 2 **C** 3 **D** 1,022 **E** 1,024

6 HARD

Each digit of a four-digit positive integer is 1, 2, 3 or 4, and digits may repeat. How many such integers are divisible by 12?

A 15 **B** 16 **C** 20 **D** 21 **E** 64

7 EXTREME

When 247 and 343 are each divided by the positive integer d, they leave the same remainder r, and $r > 5$. How many values of d are possible?

A 6 **B** 7 **C** 8 **D** 11 **E** 12

8 EXTREME

For how many integers n from 1 to 200, inclusive, is the remainder when n is divided by 6 equal to the remainder when n is divided by 8?

A 45 **B** 48 **C** 50 **D** 52 **E** 53

9 EXTREME

What is the remainder when the product of all the odd integers from 1 to 99, inclusive, is divided by 24?

A 3 **B** 9 **C** 11 **D** 15 **E** 21

10 EXTREME

When the positive integer n is divided by 4, by 6 and by 9, the sum of the three remainders is 14. What is the remainder when n is divided by 36?

A 15 **B** 17 **C** 26 **D** 33 **E** 35

Set 6 Number properties and factor counts

10 questions · target time 26:40

1 HARD

If a, b and c are integers, $a + b = c$, and ab is odd, which of the following must be true?

I. $a^2 + b^2 + c^2$ is divisible by 4.

II. c is even.

III. abc is divisible by 4.

A I only
B II only
C I and II only
D II and III only
E I, II and III

2 HARD

If a and b are nonzero integers and $\frac{a}{b} + \frac{b}{a}$ is an integer, which of the following must be true?

I. $|a| = |b|$

II. $a = b$

III. $\frac{a}{b} + \frac{b}{a} = 2$

A I only **B** II only **C** I and II only **D** I and III only **E** I, II and III

3 HARD

How many three-digit positive integers have three digits that, in some order, are three consecutive integers? (For example, 435 is one such integer, since its digits are 3, 4 and 5.)

A 15 **B** 16 **C** 42 **D** 46 **E** 48

4 HARD

If a, b and c are integers, which of the following must be even?

I. $abc + ab + bc + ca + a + b + c$

II. $(a + b)(b + c)(c + a)$

III. $a^2b + ab^2 + b^2c + bc^2 + c^2a + ca^2$

A I only
B II only
C III only
D I and II only
E II and III only

5 HARD

How many pairs of positive integers (x, y) satisfy $x^2 - y^2 - 2y = 46$?

A 1 **B** 2 **C** 3 **D** 4 **E** 6

6 HARD

The integers x, y and z satisfy $x + y + z = 0$ and $xyz = 30$. Which of the following could be the greatest of x, y and z?

I. 5

II. 6

III. 15

A I only
B I and II only
C I and III only
D II and III only
E I, II and III

7 EXTREME

Three different prime numbers $p < q < r$ are evenly spaced, so that $q - p = r - q$, and $p + q + r = 93$. How many such sets of three primes are there?

A 1 **B** 2 **C** 3 **D** 4 **E** 5

8 EXTREME

For how many integers n from 1 to 100, inclusive, is the number of odd positive factors of n odd?

A 5 **B** 7 **C** 10 **D** 14 **E** 17

9 EXTREME

Five different positive integers have a sum of 30. Which of the following must be true?

I. At least one of the integers is even.

II. The product of the integers is divisible by 4.

III. The median of the integers is at most 7.

A I only **B** II only **C** I and II only **D** I and III only **E** I, II and III

10 EXTREME

If x, y and z are nonzero numbers such that $xyz < 0$, $x + y + z > 0$ and $xy + yz + zx > 0$, which of the following must be true?

I. Exactly one of x, y and z is negative.

II. The negative number has a smaller absolute value than each of the other two numbers.

III. The absolute value of the negative number is less than half of each of the other two numbers.

A I only **B** II only **C** I and II only **D** I and III only **E** I, II and III