9 Percent Chains

College Board skill: **Percentages (Q.B)**
On test day, Problem Solving and Data is 5 to 7 of your 44 questions.

SHARE OF THE TEST

THE PATTERN | 3 KILL MOVES | 800 DRILL · 15 ITEMS

How the test builds it

Percent items at the top of the test are not about computing 15% of something. They are about translation. Three or four quantities are tied together by sentences like "is 250% greater than", "is 60% less than" and "is 0.04% of", and the question asks about two quantities that are never compared directly. The percents are chosen to break intuition: far above 100%, far below 1%, or "less than" values like 99.6% that turn a number into a crumb. Recent hard modules have carried at least one item like this on every form, and the newest version adds a twist at the answer box: the true percent is in the hundreds of thousands, so the question asks for it divided by 1,000. The other half of the family is successive and weighted change: a percent of what is left, a net change with an unknown step, two groups with different rates. The algebra is never more than one line per sentence. What costs the 800 is one word read loosely.

1 · Every phrase is a multiplier

"Is p% of" means $\times \frac{p}{100}$. "Is p% greater than" means $\times \left(1 + \frac{p}{100}\right)$. "Is p% less than" means $\times \left(1 - \frac{p}{100}\right)$.
So "240% greater than" is $\times 3.4$, "240% of" is $\times 2.4$, and "99.5% less than" is $\times 0.005$.

2 · Chain toward the asked variable

Write every quantity in terms of ONE variable, the one the question compares against. If a is 50% greater than b and c is 20% less than a:

$$c = 0.8(1.5b) = 1.2b$$

Now any question about a, b and c is a question about multiples of b.

3 · The comparison ask has a base

"A is what percent greater than B?" is

$$100\left(\frac{A}{B} - 1\right)$$

The quantity after "than" is the base. With $A = 1.2b$ and $B = b$, A is 20% greater than B, but B is about 16.7% less than A.

4 · Tiny and huge percents, and the rescaled ask

Move the decimal two places and nothing else: 0.04% is 0.0004, and 12,500% is 125. When the answer is too long for the box, the question asks for $p \div 1{,}000$ (or another power of 10). Compute p first, then divide. Don't mix the two steps.

5 · Successive and weighted changes multiply

A change after a change multiplies factors: up 20% then down 20% is $1.2(0.8) = 0.96$, a 4% loss. A percent of the remainder applies to what is left, not the original. A rate across two groups is a weighted average: each group's rate times its share.

EPIC TIP

Turn every sentence into "quantity = multiplier × quantity" before you calculate anything, and write each one in terms of the variable in the question. When the numbers are ugly, give that variable the value 100 and let the chain run.

CAUTION

Three readings cost 800s: "p% greater than" taken as "p% of", the base of a comparison taken from the wrong side of "than", and percents added or subtracted instead of multiplied. Before you bubble, reread the last sentence of the question and name the base.

LESSON 1

Chain the multipliers

Translate each sentence into a multiplier, then substitute until everything is in terms of one variable. The question's last sentence tells you which variable that is and which side of "than" is the base. Huge and tiny percents change nothing about the method; they only punish a misplaced decimal.

WORK IT TOGETHER Walk-through

Four numbers are linked. With b positive, a is 150% greater than b, c is 20% less than a, and d is 0.4% of c.

(a) Write c in terms of b.

(b) The number c is what percent greater than b?

(c) The number b is $p\%$ of d. What is the value of $\dfrac{p}{1{,}000}$?

1. "150% greater than" is $\times 2.5$: $a = 2.5b$.
2. "20% less than" is $\times 0.8$: $c = 0.8a = 0.8(2.5b) = 2b$. That is part (a).
3. Part (b): $100\left(\dfrac{c}{b} - 1\right) = 100(2 - 1) = 100$. So c is 100% greater than b.
4. "0.4% of" is $\times 0.004$: $d = 0.004c = 0.004(2b) = 0.008b$.
5. Part (c): $b = \dfrac{d}{0.008} = 125d$.
6. As a percent: b is $125 \times 100 = 12{,}500\%$ of d, so $p = 12{,}500$.
7. The rescaled ask: $\dfrac{12{,}500}{1{,}000} = 12.5$.

TRAP TO KNOW

Reading "150% greater than" as $\times 1.5$ gives $c = 1.2b$, and every later part inherits the slip. The second trap is the last step: $b = 125d$ means 12,500%, not 125%. Multiply by 100 before you rescale.

YOUR TURN

The number x is 60% less than the positive number y, and the number z is 0.5% of x. The number y is what percent of z?

DESMOS MOVE

ALGEBRA FIRST, DESMOS TO CHECK

Run the chain with the base set to 100

USE IT WHEN three or more quantities are linked by percent sentences.

EXAMPLE a is 150% greater than the positive number b, c is 20% less than a, and d is 0.4% of c. If b is $p\%$ of d, what is $\frac{p}{1{,}000}$? By hand: 12.5.

1 TYPE

```
1  b = 100
```

YOU SEE Any positive value works.

2 TYPE

```
2  a = b + 150%b
```

YOU SEE Desmos adds "of" after %. $a = 250$: "greater than" adds the percent to the whole.

3 TYPE

```
3  c = a − 20%a
```

YOU SEE $c = 200$.

4 TYPE

```
4  d = 0.4%c
```

YOU SEE $d = 0.8$.

5 TYPE

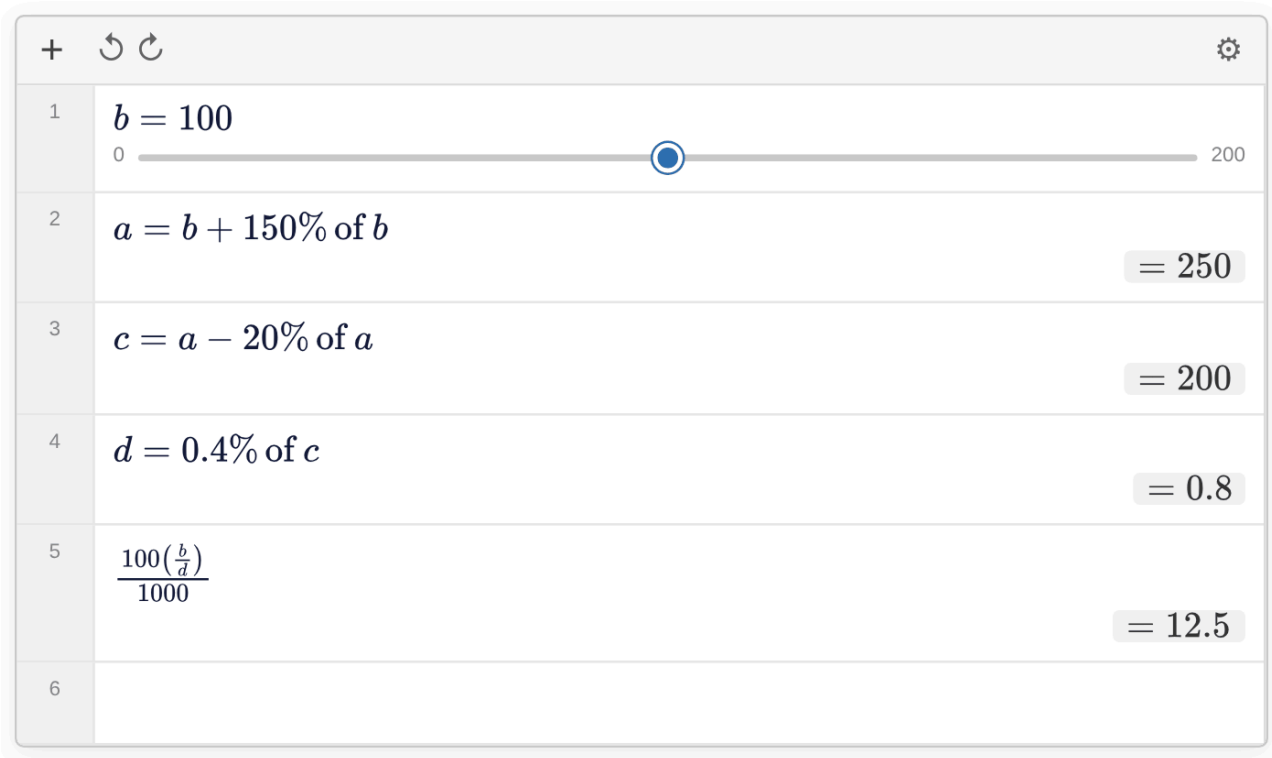
```
5  100(b/d) / 1000
```

YOU SEE Press → to leave the denominator. The line shows 12.5. That is $\frac{p}{1{,}000}$.

WHY IT WORKS

Every relation is a multiplier, so any start gives the same ratios. Desmos reads "% of"; "greater than" and "less than" are yours to translate.

YOUR DESMOS SCREEN



The last line reads 12.5, the value of $\frac{p}{1{,}000}$.

LESSON 2

Successive changes and the missing percent

When changes happen one after another, multiply the factors. When one of the changes is unknown, write the product of factors equal to the net factor and solve. If the unknown appears in two steps, the equation is a quadratic, and only one root makes sense.

WORK IT TOGETHER Walk-through

The price of a gallery membership increased by $p\%$ in 2024 and then increased by $2p\%$ in 2025. At the end of 2025 the price was 32% higher than at the start of 2024.

(a) What is the value of p?

(b) To the nearest tenth of a percent, by what percent would the price have to increase in 2026 to end 50% higher than at the start of 2024?

1. Write the factors with $r = \dfrac{p}{100}$: $(1 + r)(1 + 2r) = 1.32$.
2. Expand: $1 + 3r + 2r^2 = 1.32$, so $2r^2 + 3r - 0.32 = 0$.
3. Quadratic formula: $r = \dfrac{-3 \pm \sqrt{9 + 2.56}}{4} = \dfrac{-3 \pm 3.4}{4}$.
4. $r = 0.1$ or $r = -1.6$. A decrease of 160% is impossible, so $r = 0.1$ and $p = 10$.
5. Check: $1.1 \times 1.2 = 1.32$.
6. Part (b): the needed factor is $\dfrac{1.5}{1.32} \approx 1.1364$, an increase of about 13.6%.

TRAP TO KNOW

Adding the percents, $p + 2p = 32$, gives $p \approx 10.67$, close enough to look right and wrong in the first decimal. And in part (b), $50 - 32 = 18$ is the reflex; the second increase is measured from the 2025 price, not the original.

YOUR TURN

The number of visitors to a park decreased by 20% from 2023 to 2024 and then increased by $p\%$ from 2024 to 2025. The number of visitors in 2025 was 8% greater than in 2023. What is the value of p?

DESMOS MOVE DESMOS IS FASTER

Graph the net factor and read the percent

USE IT WHEN the unknown percent appears in more than one step, so the equation is a quadratic you would rather not solve by hand.

EXAMPLE A price increased by $p\%$ and then by $2p\%$, ending 32% higher than at the start. What is p? By hand: $p = 10$ (the root $p = -160$ is rejected).

1 **TYPE**

1 $y = \left(1 + \frac{x}{100}\right)\left(1 + \frac{2x}{100}\right)$

YOU SEE Press → after the first 100. A parabola; x stands for p, and y is the net factor.

2 **TYPE**

2 $y = 1.32$

YOU SEE A horizontal line for the net factor.

3 **ZOOM**

Tap the wrench and set x from -200 to 50 and y from -1 to 3.

YOU SEE The line crosses the parabola twice.

4 **TAP**

Tap both intersection points.

YOU SEE Labels $(-160, 1.32)$ and $(10, 1.32)$.

5 **READ**

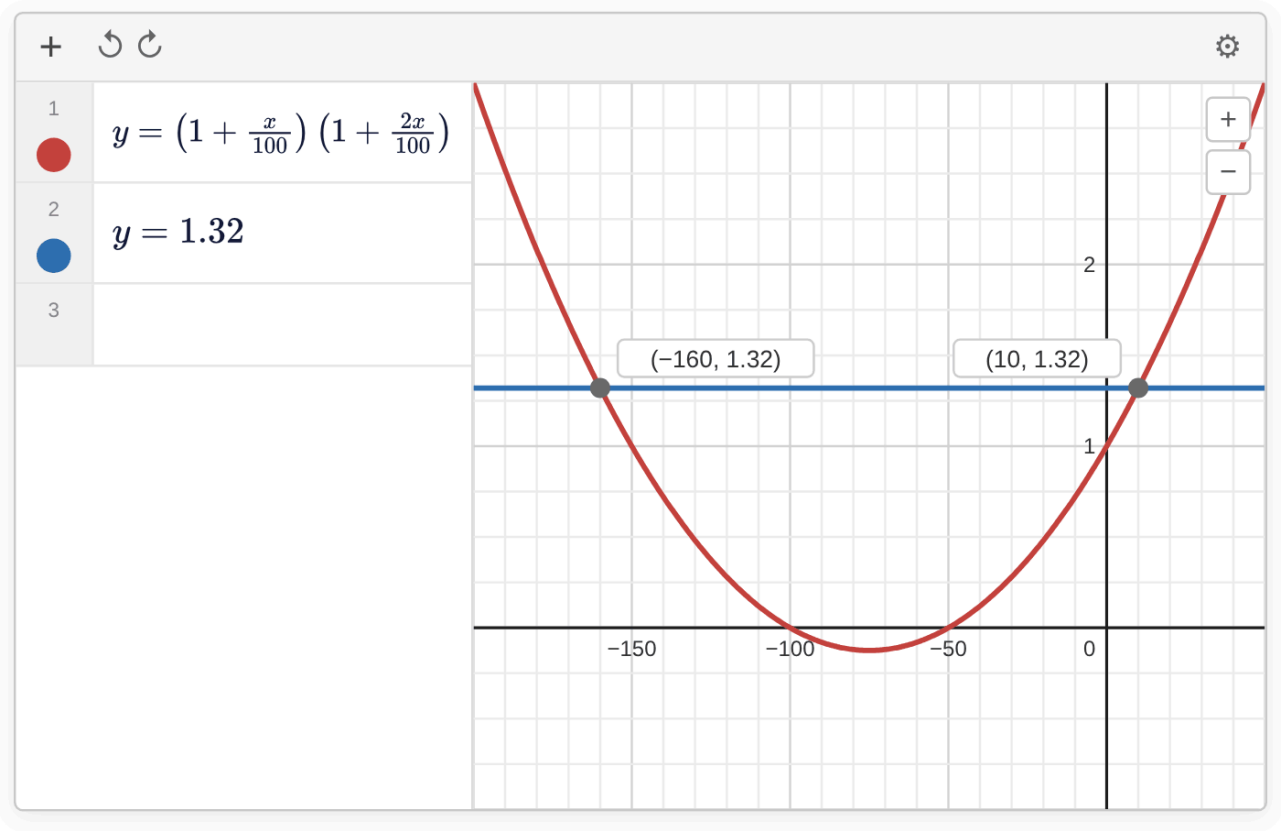
Keep the root that is a real change.

YOU SEE -160 would be a drop of more than 100%, so $p = 10$.

WHY IT WORKS

Each crossing is a value of p whose two factors multiply to the net factor. The graph finds both roots at once; the context throws one out.

YOUR DESMOS SCREEN



Two crossings, $p = -160$ and $p = 10$; only $p = 10$ is a real change.

LESSON 3

Weighted percents and the percent of what is left

When a whole splits into parts with different rates, the overall rate is a weighted average: share times rate, added. When a percent is taken "of the rest" or "of the remainder", it applies to what is left after the earlier steps. Set the whole to 100 and both kinds become arithmetic.

WORK IT TOGETHER Walk-through

A farm's land is 55% cropland and 45% pasture. Of the cropland, 20% is fenced, and of the pasture, 60% is fenced.
(a) What percent of the farm's land is fenced?
(b) To the nearest whole percent, what percent of the fenced land is pasture?
(c) To the nearest whole percent, the fenced cropland is what percent less than the fenced pasture?

1. Let the farm have 100 acres: 55 acres of cropland and 45 acres of pasture.
2. Fenced cropland: $0.2(55) = 11$ acres. Fenced pasture: $0.6(45) = 27$ acres.
3. Part (a): $11 + 27 = 38$ acres of 100, so 38%.
4. Part (b): $\frac{27}{38} \approx 0.7105$, so about 71%.
5. Part (c): $100\left(1 - \frac{11}{27}\right) \approx 59.3$, so about 59% less.
6. Check part (a) as a weighted average: $0.55(20\%) + 0.45(60\%) = 11\% + 27\% = 38\%$.

TRAP TO KNOW

Averaging the two rates, $\frac{20 + 60}{2} = 40$, ignores that there is more cropland than pasture. In part (b) the base is the fenced land, 38, not the pasture, 45.

YOUR TURN

In a school, 30% of the students in section X and 55% of the students in section Y passed an exam. Overall, 40% of the students in the two sections passed. Section Y has 60 students. How many students are in section X?

DESMOS MOVE

ALGEBRA FIRST, DESMOS TO CHECK

Pick 100 and let the shares run

USE IT WHEN a whole is split into parts with different percents, and the question asks for an overall percent or one part's share of a total.

EXAMPLE A farm's land is 55% cropland and 45% pasture; 20% of the cropland and 60% of the pasture is fenced. (a) What percent of the land is fenced? (b) What percent of the fenced land is pasture? By hand: (a) 38%; (b) about 71%.

1 TYPE

```
1  L = 100
```

YOU SEE The whole farm, 100 acres.

2 TYPE

```
2  C = 55%L
```

YOU SEE Desmos adds "of" after %. $C = 55$, the cropland.

3 TYPE

```
3  F = 20%C + 60%(L - C)
```

YOU SEE $F = 38$: part (a), 38% of the land is fenced.

4 TYPE

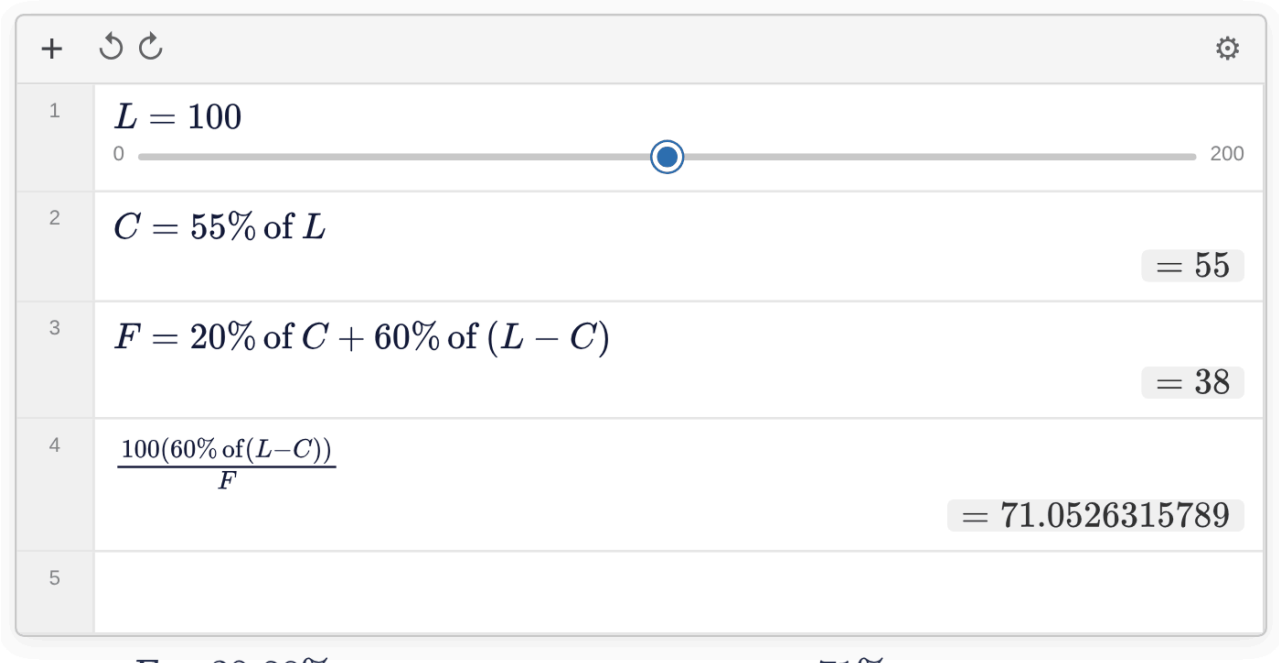
```
4  100(60%(L-C))/F
```

YOU SEE The line shows $71.0526\ldots$: part (b), about 71%.

WHY IT WORKS

With the whole set to 100, every amount is also a percent of the whole, and the last line divides by the right base, the fenced land. Changing L to any other number leaves both answers the same.

YOUR DESMOS SCREEN



$F = 38$: 38% of the land is fenced, and about 71% of that is pasture.

800 Drill

800 LEVEL

15 items · Desmos allowed · Hard to hardest

1

Q.B

For a positive number q, the numbers p, r, and s satisfy three conditions: p is 20% less than q, r is 150% greater than p, and s is 0.4% of r. Which of the following expressions is equivalent to $q + s$?

A) $0.504r$

B) $0.54r$

C) $0.837r$

D) $2.004r$

2

Q.B

The positive number B is 400% greater than the positive number C, and the number A is 0.016% of B. If C is $p\%$ of A, what is the value of $\frac{p}{1{,}000}$?

GRID-IN

3

Q.B

At the start of a year, quantity A was 36% less than quantity B. During the year, A increased by $p\%$ and B increased by 60%, and at the end of the year the two quantities were equal. What is the value of p?

A) 96

B) 117.6

C) 150

D) 250

4

Q.B

A farm sold 30% of its apple harvest at a market and sent 40% of the remaining apples to a cannery. It then donated 25% of the apples it still had and stored the rest. The farm sent 1,260 more kilograms of apples to the cannery than it donated. How many kilograms of apples did the farm store?

A) $2{,}016$

B) $2{,}268$

C) $2{,}940$

D) $7{,}200$

5

Q.B

The salt concentration of solution A is 60% less than the salt concentration of solution B. How many liters of solution B must be mixed with 3 liters of solution A to make a mixture whose salt concentration is 25% less than the salt concentration of solution B?

GRID-IN

6

Q.B

The number a is 60% greater than the positive number b, and the number c is 37.5% less than a. Which of the following must be true?

I. a is 37.5% greater than c.
II. b is 37.5% less than a.

A) I only

B) II only

C) I and II

D) Neither I nor II

7

Q.B

A store sets the regular price of a jacket 60% greater than the store's cost for the jacket. During a sale, the store lowers the regular price by $p\%$. At the sale price, the store's profit on the jacket (sale price minus cost) is 20% of the sale price. What is the value of p?

GRID-IN

8

Q.B

In a group of hikers, 25% of the adults and 40% of the teenagers have hiked the trail before. Of all the hikers in the group, 30% have hiked the trail before. The group consists only of adults and teenagers. The number of adults is what percent greater than the number of teenagers?

A) 33.3%

B) 50%

C) 66.7%

D) 100%

9

Q.B

The price of a train pass increased by $p\%$ and then decreased by $2p\%$, where p is positive. The final price was 12% less than the original price. What is the value of p?

A) 10

B) 12

C) 20

D) 60

10

Q.B

The positive number y is 99.6% less than the number x, and the number z is 2,400% greater than y. The number z is what percent of x?

A) 0.4%

B) 9.6%

C) 10%

D) $2{,}490\%$

11

Q.B

From 2023 to 2024, the percent of a town's residents who owned an electric car increased by 2.4 percentage points, which was a 40% increase in that percent. Over the same period, the town's population increased by 5%. The number of residents who owned an electric car increased by $p\%$ from 2023 to 2024. What is the value of p?

GRID-IN

12

Q.B

The positive numbers a, b, and c are related as follows: a is $k\%$ greater than b, and b is $k\%$ less than c, where $0 < k < 100$. Which of the following must be true?

A) $a < c$

B) $a = c$

C) $a > c$

D) There is not enough information to compare a and c.

13

Q.B

The side length of the square base of a right rectangular prism is increased by 50%, and the height of the prism is decreased by $p\%$. The volume of the new prism is 35% greater than the volume of the original prism. What is the value of p?

A) 10

B) 15

C) 32.5

D) 40

14

Q.B

The population of town P is 0.5% of the population of a region. The population of town Q is 300% greater than the population of town P. The population of the region is what percent of the combined population of towns P and Q?

A) 2.5%

B) 400%

C) $4{,}000\%$

D) $5{,}000\%$

15

Q.B

Last year, 60% of a store's revenue came from in-store sales and the rest came from online sales. This year, revenue from in-store sales decreased by 15% and revenue from online sales increased by $p\%$. The store's total revenue this year was 3% greater than last year. What is the value of p?

GRID-IN